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Effect of linear and quadratic coupling on dynamical parameters of an optomechanical oscillator

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Abstract. *Dynamics of microcantilevers are of importance in micro-mechanical systems to enhance the functionality and applicable range of the cantilevers in vibration transducing and highly sensitive measurement. In this study, using the semi-classical Hamiltonian formalism, we study in detail the modification of the mechanical frequency and damping rate taking into account both the linear and quadratic coupling between the mechanical oscillator and the laser field in an optomechanical system. We have shown that, the linear coupling greatly enhances the modification of the effective mechanical frequency and the effective damping rate while the quadratic coupling reduces these quantities. For a MHz-frequency oscillator, the damping rate could be 10^5 times increased and the frequency is several times modified. These results help clarifying the origin of the modification of the susceptibility function for cooling of the mechanical mode.*

Keywords: microcantilever, mode shape, analytical method, overhang-shaped, T-shaped.

Classification numbers: 07.79.Lh, 78.20.N-, 65.40.De.

1. Introduction

Light-matter coupling at macroscopic and mesoscopic scales has garnered significant interest over the past two decades to elucidate quantum-to-classical physics. In the experiment for seeking the gravitational waves from the universe, a test mass can be controlled to reveal the extremely subtle impact of gravitational waves on Earth [1–4]. In the experiment for revealing the quantum-to-classical transition, the pico-gram mass oscillators were used and their effective temperature could be reduced to the millikelvin range, where the phonon number approaches zero [5–8]. In such systems, reducing the noise from environmental sources, the quantum properties of the driving light, or the mutual interaction between light and object is crucial. A system comprising two mirrors that traps and amplifies an electromagnetic field (a laser) has proven highly effective in this domain. When the distance between the two mirrors, the cavity length L_c, is close to a multiple of the laser wavelength, a small change in the cavity length reduces the stored field and the radiation force exerted on the mirror. As a result, the coupling between the mechanical and optical modes is maintained.

Beside the setting where the linear optomechanical coupling exists, the system with second-order coupling is also available for specific arrangement, for example, membranes (such as graphene layer, semi-transparent thin film) inserted inside an optical cavity by Harris *et al.* [7, 9], Vitali et. al. [10], Purdy *et al.* [11], Favero *et al.* [12, 13], and Weig *et al.* [14]. Locating at the anti-node of the field intensity inside the optical cavity, the oscillator suffers to a second-order dependent coupling to the field intensity. For a detailed discussion on the system settings and quadratic coupling of these systems, the reader is referred to the work of Favero et al. [13]. The quadratic coupling gives fruitful contribution to the cooling in optomechanics.

In this study, we examine the contribution of both the linear and the quadratic coupling on the modification of the mechanical frequency and the damping rate of the oscillator in an optomechanical system. These two parameters are crucial in controlling of the dynamics of the oscillator and any change of them could lead directly to the modification of the susceptibility function, the function that determines the effective temperature and oscillating (mechanical) energy. Therefore, a detailed study of the modification of the frequency and damping rate is of interest.

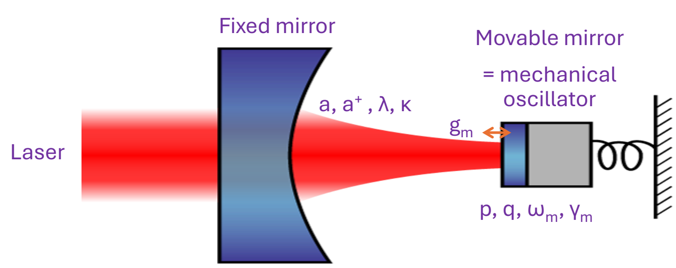

Fig. 1. The model of an optomechanical system. The optical cavity is formed by setting two reflective mirrors made of metallic thin film. One mirror is fixed and the other is movable. The movable mirror could be mounted on a singly or doubly clamped mechanical cantilever and becomes a mechanical oscillator with a resonance frequency ω_m, a damping rate γ_m and the momentum[position] variable p[q]. The cavity is irradiated by a laser beam of wavelength λ with the creation/annihilation operator $a^\dagger/a$, a decay rate κ.

The optomechanical system including an optical microcavity irradiated by a laser is shown in Fig. 1. The first mirror is a semi-transparent thin film, and the second mirror plays the role of a mechanical oscillator, which is movable and has a mechanical frequency of ω_m, a decay/damping rate of γ_m, and located at L_C from the first mirror. The thicknesses of the two films were chosen so that they could maximize the effect of the radiation pressure on the two inner surfaces [15–18]. The mirrors could be made of or coated by metallic thin films that is semi-transparent, i.e. a part of the light is transmitted and the other part is reflected. The thickness of the first mirror is usually smaller than that of the second mirror. In a previous study [16], we have shown that the thickness of the first mirror should range from 20–60 nm where that of the second mirror is 40–100 nm. The cavity length L_C is chosen so that it approximately equals to a number of the optical wavelength, $L_C \simeq n\lambda/2$ where n is an integer. A small displacement of the second mirror gives rise to a reduction in the radiation pressure on it and increases the restoring force, which pulls the mirror back to its initial position. This mutual action is represented by the couplings g_m and g_2. In this study, the thicknesses are assumed to greatly enhance the mutual coupling between the laser light and the mirrors, and is represented by the optomechanical couplings g_m and g_2.

2. Hamiltonian formalism for optomechanical oscillators

From the Hamiltonian of the mechanical oscillator, $H_m = (1/2)(P^2/m + m\omega_m^2 Q^2)$ where P and Q are the momentum and position variables, we make a variable change as $p = P/\sqrt{m\hbar\omega_m}$ and $q = Q\sqrt{m\omega_m/\hbar}$. The coupling between the optical and the mechanical mode is assumed to involve the linear and quadratic [19–21] terms as

$$H_i = -\hbar g_m a^\dagger a q + \hbar g_2 a^\dagger a q^2, \tag{1}$$

where $g_m = \frac{\partial\omega_c}{\partial x} x_{ZPF}$ is the linear optomechanical coupling strength, and $\frac{\partial\omega_c}{\partial x} \simeq \frac{\omega_c}{L_c}$, which leads to $g_m = \frac{\omega_c}{L_c} x_{ZPF}$ where $x_{ZPF} = \sqrt{\frac{\hbar}{2\omega_m m}}$ is the zero point fluctuation [22], the quantity that estimates the vibration amplitude of an oscillator when its effective temperature T_{eff} is in the order of a quantum $k_B T_{eff} \leq \hbar\omega$. The optomechanical coupling Hamiltonian says that the coupling is linearly proportional to the stored laser intensity, $\propto a^\dagger a$, and the small displacement q, and that in the quadratic term is quadratic dependent on displacement, q^2.

As a result, we obtain the total Hamiltonian (in the rotating frame of the pump laser with frequency $\omega_p = 2\pi c/\lambda$) where the laser has a power of P_i, an amplitude of $\epsilon_p = [P_i\kappa/(2\hbar\omega_p)]^{1/2}$ [22], and a cavity damping rate of κ

$$H = \hbar\Delta_0 a^\dagger a + \frac{1}{2}\hbar\omega_m(p^2+q^2) - \hbar g_m a^\dagger a q + \hbar g_2^\dagger a q^2 + i\hbar\epsilon_p(a^\dagger - a), \tag{2}$$

where $\Delta_0 = \omega_c - \omega_p$ and $\omega_c = 2\pi c/L_c$ is the frequency of the photon mode inside the cavity. Using the Heisenberg equations for the operators a, $a^\dagger$, q, and p, such as $-i\hbar\dot{a} = [a, H]$,

$$\dot{a} = -i\Delta_0 a + i g_m a q - i g_2 a q^2 + \epsilon_p, \tag{3a}$$

$$\dot{a}^\dagger = i\Delta_0 a^\dagger - i g_m a^\dagger q + i g_2 a^\dagger q^2 + \epsilon_p, \tag{3b}$$

$$\dot{p} = -\omega_m q + g_m a^\dagger a - 2 g_2 a^\dagger a q, \tag{3c}$$

$$\dot{q} = \omega_m p, \tag{3d}$$

then adding the terms of noises and the dampings, we obtained the following equations,

$$\dot{a} = -(\kappa + i\Delta_0)a + i g_m a q - i g_2 a q^2 + \epsilon_p + \sqrt{2\kappa} a_{in}, \tag{4a}$$

$$\dot{a}^\dagger = -(\kappa - i\Delta_0)a^\dagger - i g_m a^\dagger q + i g_2 a^\dagger q^2 + \epsilon_p + \sqrt{2\kappa} a_{in}^\dagger, \tag{4b}$$

$$\dot{p} = -\gamma_m p - \omega_m q + g_m a^\dagger a - 2 g_2 a^\dagger a q + \xi(t), \dot{q} = \omega_m p. \tag{4c}$$

The cavity mode is affected by the input noise from the vacuum radiation a_{in} and the mechanical mode is affected by the fluctuation ξ. They follow the correlation functions [23–26]

$$\langle a_{in}(t)a_{in}^{\dagger}(t')\rangle = [N(\omega_c)+1]\delta(t-t'), \tag{5a}$$

$$\langle a_{in}^{\dagger}(t)a_{in}(t')\rangle = N(\omega_c)\delta(t-t'), \tag{5b}$$

$$\langle a_{in}(t)a_{in}(t')\rangle = \langle a_{in}^{\dagger}(t)a_{in}^{\dagger}(t')\rangle = 0, \tag{5c}$$

$$\langle \xi(\omega)\xi(\omega')\rangle = \frac{\gamma_m}{\omega_m}\omega\left[\coth\left(\frac{\hbar\omega}{2k_BT}\right)+1\right]\delta(\omega+\omega'), \tag{5d}$$

where N is the two-photon correlation functions [24]. The steady state solutions a_s and q_s are obtained by setting the derivatives to zero, e. g. at $\dot{a}=0$ we get a_s, at $\dot{q}=0$ we get q_s and we have

$$\begin{cases} a_s = & \dfrac{\epsilon_p}{\kappa+i(\Delta_0-g_mq_s+g_2q_s^2)} \\ a_s^{\dagger} = & \dfrac{\epsilon_p}{\kappa-i(\Delta_0-g_mq_s+g_2q_s^2)} \\ q_s = & g_m|a_s|^2/(\omega_m+2g_2|a_s|^2), p_s=0. \end{cases} \tag{6}$$

To obtain the fluctuation spectra of the transmitted field, we linearize the quantum Langevin equation by writing the operators as the summation of their mean values, and the fluctuation operators [27], such as $a=a_s+\delta a$. We then keep the linear terms and skip all terms that is higher than second order of fluctuations, such as δa^2 or $\delta a\delta q$. Thus, we obtain,

$$\delta\dot{a} = -(\kappa+i\Delta)\delta a + iG_a\delta q + \sqrt{2\kappa}a_{in}, \tag{7a}$$

$$\delta\dot{a}^{\dagger} = -(\kappa-i\Delta)\delta a^{\dagger} - iG_a^*\delta q + \sqrt{2\kappa}a_{in}^{\dagger}, \tag{7b}$$

$$\delta\dot{p} = -\gamma_m\delta p - (\omega_m+2g_2|a_s|^2)\delta q + G_a^*\delta a + G_a\delta a^{\dagger} + \xi, \tag{7c}$$

$$\delta\dot{q} = \omega_m\delta p, \tag{7d}$$

where $\Delta=\Delta_0-g_mq_s+g_2q_s^2$ is the corrected detuning and $G_a=(g_m-2g_2q_s)a_s$. Taking the Fourier transform $\mathscr{F}[\delta\dot{a}(t)]\to -i\omega\delta a(\omega)$, Eq. (7) could be rewritten in a matrix form as follow,

$$\begin{pmatrix} -i\omega+\kappa+i\Delta & 0 & 0 & -iG_a \\ 0 & -i\omega+\kappa-i\Delta & 0 & iG_a^* \\ -G_a^* & -G_a & -i\omega+\gamma_m & \omega_m+2g_2|a_s|^2 \\ 0 & 0 & -\omega_m & -i\omega \end{pmatrix}\begin{pmatrix} \delta a \\ \delta a^{\dagger} \\ \delta p \\ \delta q \end{pmatrix} = \begin{pmatrix} \sqrt{2\kappa}a_{in} \\ \sqrt{2\kappa}a_{in}^{\dagger} \\ \xi \\ 0 \end{pmatrix}. \tag{8}$$

Assuming that the Routh-Hurwitz criterion for the parameters is satisfied, then Eq. (8) has solutions [28]. From Eq. (6), we could choose the relative phase reference for the intracavity field and the external laser so that a_s is real and positive, for example,

$$\epsilon_p = |\epsilon|e^{-i\theta} = |\epsilon|\frac{\kappa+i(\Delta_0-g_mq_s+g_2q_s^2)}{\sqrt{\kappa^2+(\Delta_0-g_mq_s+g_2q_s^2)^2}},$$

we denote $G_a^*=G_a=G$ as the reduced coupling strength. The solution of Eq. (8) is

$$\delta q(\omega) = \frac{-\omega_m}{d(\omega)}\left\{\left[\Delta^2+(\kappa-i\omega)^2\right]\xi - iG\sqrt{2\kappa}\left[(\omega+i\kappa-\Delta)a_{in}^{\dagger}+(\omega+i\kappa+\Delta)a_{in}\right]\right\}, \tag{9}$$

$$\delta p(\omega) = (-i\omega)(\omega_m)\delta q(\omega), \tag{10}$$

and δa and $\delta a^{\dagger}$ that are not shown here for brevity and because we are concentrating on the effect of the opto-mechanical coupling on the phonon variance only. In Eqs. (9) and (10),

$$d(\omega) = 2\Delta G^2\omega_m + [(\omega + i\kappa)^2 - \Delta^2][\omega_m(\omega_m + g_2) - \omega^2 - i\omega\gamma_m]. \tag{11}$$

Dividing the denominator and numerator on the rhs. of Eq. (11) to $\Theta = (\omega + i\kappa - \Delta)(\omega + i\kappa + \Delta)$ and letting let $\frac{G^2\omega_m}{\Theta} = \Xi$, we obtain

$$\begin{aligned} d_\Theta(\omega) =& d(\omega)/\Theta = \omega_m^2 - \omega^2 - i\omega\gamma_m + 2\Delta\Xi = \omega_m^2 - \omega^2 - i\omega\gamma_m + 2\Delta(Re[\Xi] + i.Im[\Xi]) \\ =& \omega_{eff}^2 - \omega^2 - i\omega\gamma_{eff}. \end{aligned} \tag{12}$$

Finally, the effective mechanical frequency and the effective damping are written as

$$\omega_{eff}^2(\omega) = \omega_m^2 + 2\Delta.Re[\Xi] = \omega_m^2 + G^2\omega_m \frac{2\Delta(\omega^2 - \Delta^2 - \kappa^2)}{[(\omega-\Delta)^2 + \kappa^2](\omega+\Delta)^2 + \kappa^2]}, \tag{13a}$$

$$\gamma_{eff}(\omega) = \gamma_m + 2\Delta.Im[\Xi] = \gamma_m + G^2\omega_m\kappa \frac{4\Delta}{[(\omega-\Delta)^2 + \kappa^2](\omega+\Delta)^2 + \kappa^2]}. \tag{13b}$$

We have obtained the analytical formula for the mechanical frequency and the damping rate of the oscillator, which depend on the coupling strength G, the de-tuning Δ, and the linear and quadratic couplings.

3. Results and Discussion

To present the results, we used these parameters: $\omega_m = 2\pi\times10^6$ Hz is the mechanical frequency, $\gamma_m = 2\pi\times260$ Hz, $m = 5$ ng is the mass of the mechanical oscillator. The laser to detune and drive the mechanical oscillator has a wavelength of $\lambda = 1064$ nm, a decay rate $\kappa = 6\pi\times10^6$ Hz, and an input power $P_i = 0.1$–10 mW. This power P_i gives rise to ϵ_p in Eq. (2) and also the steady-state values of a_s and q_s in Eq. (6). As a result, the coupling strength G_a (or G) depends on P. Solving Eq. (6) for a_s or q_s, multiple solutions are obtained which implies the bistability of the system, that is, there are three values of a_s for an input power P_i. In this study, we choose the input power P_i so that the coupling G is within the range [0, G_m] where G_m is the maximum value at which the oscillator starts to fall into the bistability region. For the quadratic coupling strength, we limit our research in a qualitative regime where g_2 is much smaller than G, $g_2 \sim 10^{-3}g_m$ [21]. The maximum value of g_2 is chosen to be $0.006G_m$.

In Fig. 2(left), we present the change of the effective mechanical frequency versus the optomechanical coupling strength G. We could see that ω_{eff} is significantly changed versus G. For increasing G, ω_{eff} increases for $\omega > 1$ and decreases for $\omega < 1$. At $\omega = \omega_m$, $\omega_{eff} = \omega_m$. It is worth noting that these modifications will directly change the susceptibility function $\chi(\omega)$ of the mechanical oscillator under the exertion of external noise, e.g. from the thermal noise. From the equation of motion of the oscillator, the variance of the position could be expressed as

$$\langle x^2(\omega)\rangle = \chi(\omega)F_{noise}(\omega), \tag{14}$$

where $F_{noise}(\omega)$ here denotes the total effects of external noise from both the photonic and bosonic modes. As a result, the oscillation amplitude is modified. If one reduces the value of the function $\chi(\omega)$, the oscillator will reduce the effects from the environment and could be significantly cooled. Karrai *et al.* [29] used such a mechanism to cool a microcantilever from room temperature to 18

K using a milliwatt laser source. In the right panel, the effective mechanical damping rate γ_{eff} is shown. γ_{eff} here is significantly increased, up to 10^5 times the original damping rate of several tens of Hz. Adapting to the effective mechanical frequency, this enhanced damping contributes to the modification of the susceptibility function.

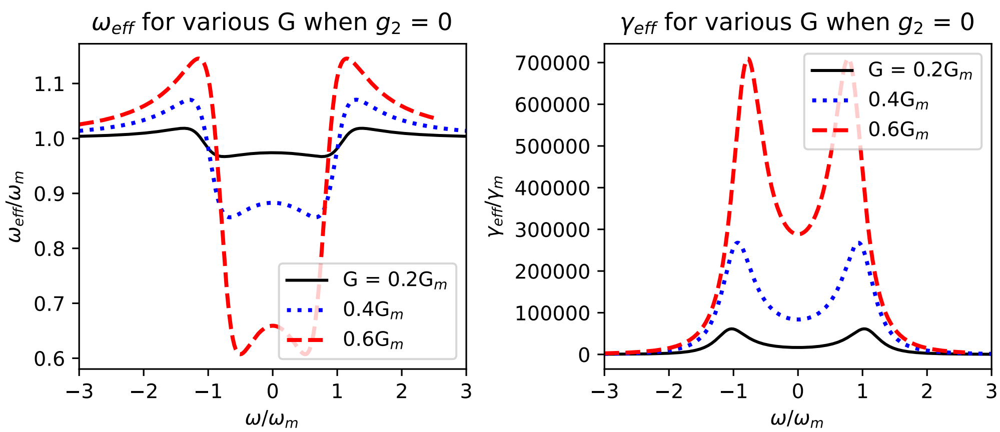

Fig. 2. (left) Normalized effective mechanical frequency ω_{eff}/ω_m and (right) effective damping rate γ_{eff}/γ_m versus the reduced optomechanical coupling strength G. Based on Eq. (13), ω_{eff} and γ_{eff} are symmetric versus the origin and have extrema around $\omega_m \pm \Delta$. G_m is the maximal coupling strength that the oscillator starts coming to the bistability region. Great modifications of frequency and damping lead to changes in the susceptibility function $\chi(\omega)$ [see Eq. (14)], the mechanism of the passive laser cooling.

The coupling strength of the quadratic coupling is dependent on the system setting and is ususlly in the order of 10^{-6}–10^{-3} of the linear coupling. In the work of Xuereb and Paternostro [30], g_2 is chosen to be $\simeq 2\pi\times 10$ μHz while $g_m = 2\pi\times$ 36 Hz for 1 MHz oscillator, i.e. $g_2/g_m \simeq 10^{-6}$ or He *et al.* [31] also used $g_2/g_m \simeq 10^{-6}$, while Ghorbani *et al.* [21] used $g_2/g_m \simeq 10^{-3}$.

In Fig. 3, contribution of the second-order coupling is estimated. As we could see, the coupling g_2 could lead to a decrease in the modification of the effective mechanical frequency and the damping rate. This arises from the plus sign in the Hamiltonian term of the second-order process [Eq. (2)]. The second-order process could arise as a higher term in the Maclaurin series of the optomechanical interaction [28] or naturally arise in a specific setting up of the system where the second-order process is excited; for example, the oscillator is inserted in the anti-node of an empty optical microcavity [21, 32]. Nevertheless, the appearance of a membrane in the middle of the empty cavity usually redistribute the field intensity inside the cavity. The vibration of the oscillator is a complex superposition of multiple higher-order modes, where the amplitude of these modes can be tuned for great amplification. We leave the study of this effect for future work.

4. Conclusion

In this study, we have investigated the modification of the mechanical frequency and damping rate in an optomechanical system using the semi-classical Hamiltonian formalism. By considering both linear and quadratic couplings between the mechanical oscillator and the laser field, we

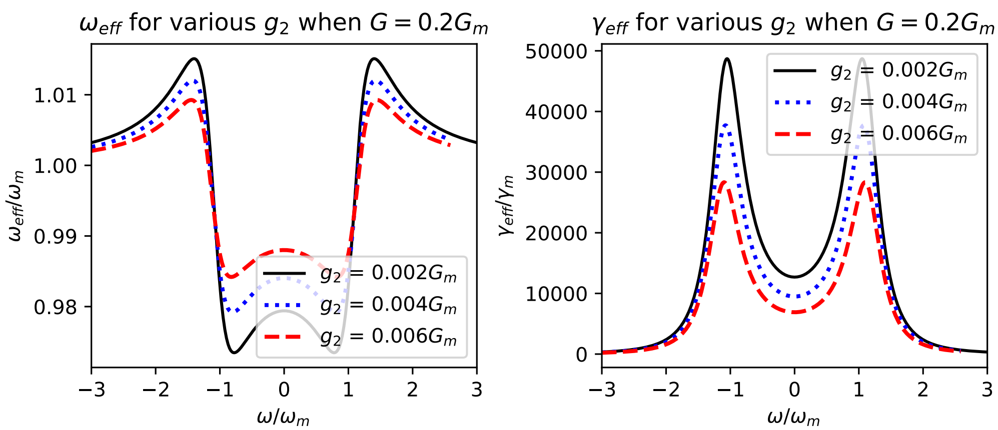

Fig. 3. (left) Normalized effective mechanical frequency ω_{eff} and (right) effective damping rate γ_{eff} versus the quadratic coupling strength g_2 for a fixed value of linear coupling G. The frequency and damping rate have been reduced due to the plus sign, in contrast to the minus sign, of the Hamiltonian term in the total Hamiltonian describing the optomechanical coupling.

have shown that linear coupling significantly enhances the effective mechanical frequency shift and damping rate, whereas quadratic coupling mitigates these effects. For a MHz-frequency oscillator, the damping rate can be increased by a factor of 10^5, while the frequency experiences notable modifications. These findings provide deeper insights into the role of optomechanical interactions in tuning the susceptibility function, which is crucial for applications such as mechanical mode cooling and precision measurements.

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